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The Faraday effect of an antiferromagnetic photonic crystal with a defect layer

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Abstract
A theoretical calculation of the Faraday optical rotation effect of an antiferromagnetic (AF) photonic crystal is presented. This crystal is composed of AF and dielectric (D) layers and contains an AF defect layer. From the theoretical results for the FeF_2–SiO_2 crystal, we see a defect mode with high transmission and a high Faraday rotation angle in the optical stop band for $\omega/2\pi c < 105\,\mathrm{cm}^{-1}$. The Faraday rotation of the mode is about 28° mm^{-1} and 15 times that of the single AF film. Another more striking property is that the rotation in the vicinity of the zero-field AF resonance frequency is even larger than that of the defect mode: about 250 times. The Faraday rotation can be tuned by changing the strength of the external static magnetic field.

1. Introduction

Magnetic materials hold promise as prospective components of magnetic photonic crystals (MPCs) since they allow tuning of their optical properties under dc magnetic field action or with temperature change [1]. The very high Faraday rotation in these crystals was proven experimentally [2–4]. The application of ferrite photonic crystals in a microwave PC multiplexer was realized by Kee *et al* [5]. In the previous works, the materials used as magnetic layers in the crystals were magneto-optical or ferromagnetic, such as Bi:DyIG [2], Bi:YIG [4], Ce:YIG [6] and Co ferrite [7], possessing the optical rotation effect from the dielectric coefficient tensor [2, 8] for wavelengths in the range of visible light. However if one is interested in processing infrared signals, an antiferromagnetic (AF) medium with the resonant frequency in the infrared range may be selected, and simultaneously its optical rotation results from the AF permeability tensor. In the references published previously, we have found no studies of AF photonic crystals (AFPC). This may be because, in the absence of a static magnetic field, an AF film does not possess the gyromagnetic ability, and even in the presence of such a field it has, generally speaking, just a small gyromagnetic action. However, as we know, some new properties have been unexpectedly found from artificial structures composed

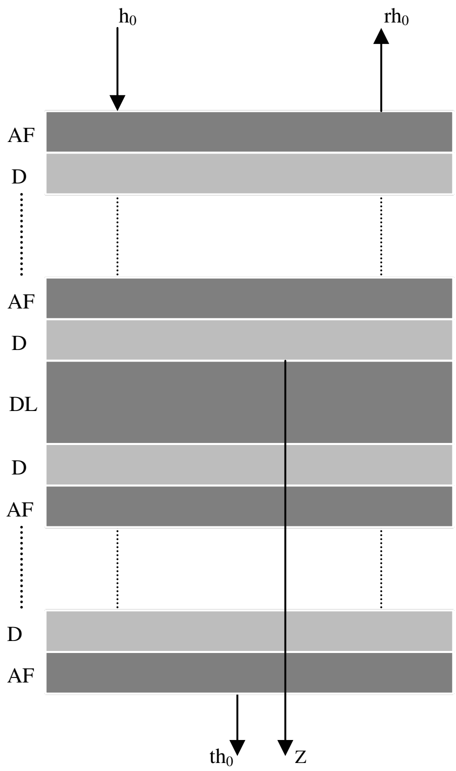

Figure 1. AFPC composed of AF, D and one DL layers, where AF, D and DL represent antiferromagnetic, dielectric and defect layers, respectively. An infrared radiation is incident normally on the upper surface of this system. The z axis is parallel to the external field H_0 and the sublattice magnetization.

of different medium layers [9, 10]. Thus one may wonder how the transmission spectrum and Faraday effect fare if the magnetic layers in the MPCs are changed into AF layers. In this paper, we shall present the Faraday rotation and transmission spectrum of an AFPC with a defect layer.

2. Expressions for transmission and rotation

An AFPC with a defect layer (DL) is illustrated in figure 1. The DL layer is the same as the other AF layers, except as regards its thickness. The sublattice magnetizations (M_0) of the AF layers and the external magnetic field (H_0) are pointed along the z axis, whose origin is on the upper surface of the defect layer. We arrange N bilayers above and N bilayers below the defect layer, where each bilayer contains an AF layer with thickness d_a and dielectric constant ε_a and a dielectric (D) layer with thickness d_d and dielectric constant ε_d, so a one-bilayer thickness is equal to $L = d_a + d_d$. The thickness of the defect layer is taken as $d = 2d_a$. The AF layers have the magnetic permeability tensor μ and that of the dielectric layers is 1. Electromagnetic radiation with the magnetic field h_0 is incident normally on the upper surface of the AFPC, producing a reflection field rh_0 and transmission field th_0. In this geometry, the right- and left-handed circular polarization radiations (hereafter, they are indicated by subscripts + and −) moving in the z direction are two eigenmodes of the wave equations in the AFPC. Then the two circularly polarized waves correspond to the AF permeabilities μ_+ and μ_- whose expressions are

$$\mu_+ = 1 + \frac{2\gamma^2 H_a 4\pi M_0}{\omega_r^2 - (\omega + i\tau + \gamma H_0)^2} \qquad \text{and} \qquad \mu_- = 1 + \frac{2\gamma^2 H_a 4\pi M_0}{\omega_r^2 - (\omega + i\tau - \gamma H_0)^2}, \tag{1}$$

with H_e and H_a meaning the exchange and anisotropy fields, but $\omega_r = \gamma\sqrt{(2H_eH_a + H_a^2)}$, the zero-field resonance frequency. τ is introduced as a damping parameter and γ the gyromagnetic ratio. Here we see two different resonance frequencies $\omega_1 = \omega_r - \gamma H_0$ and $\omega_2 = \omega_r + \gamma H_0$ from expressions (1). Applying the boundary conditions of the wave fields h_x and $e_y = \frac{i}{\varepsilon_0 \varepsilon \omega} \frac{\partial h_x}{\partial z}$ being continuous at the interfaces and the transfer matrix method [13], we find that the transmission and reflection waves satisfy the relation

$$\begin{bmatrix} h_0 r \\ h_0 \end{bmatrix} = \Lambda \begin{bmatrix} h_0 t \\ h_0 t \end{bmatrix} \tag{2}$$

with

$$\Lambda = \frac{1}{16\Delta\Delta_{01}} \begin{pmatrix} \delta_\pm(\Delta_{01}+1) & \delta_\pm^{-1}(\Delta_{01}-1) \\ \delta_\pm(\Delta_{01}-1) & \delta_\pm^{-1}(\Delta_{01}+1) \end{pmatrix} T^{N-1} \begin{pmatrix} \delta_d(\Delta+1) & \delta_d^{-1}(\Delta-1) \\ \delta_d(\Delta-1) & \delta_d^{-1}(\Delta+1) \end{pmatrix}$$
$$\times \begin{pmatrix} \delta_\pm(\Delta+1) & \delta_\pm^{-1}(1-\Delta) \\ \delta_\pm(1-\Delta) & \delta_\pm^{-1}(\Delta+1) \end{pmatrix} T'^{N-1} \begin{pmatrix} 1-\Delta_{01} & 0 \\ 0 & \Delta_{01}+1 \end{pmatrix}, \tag{3}$$

in which T or T' is the transfer matrix indicated by

$$T'_{11} = T'^{*}_{22} = \delta_d \left[\cos(k_\pm d) + \frac{i(1+\Delta^2)}{2\Delta} \sin(k_\pm d) \right] \tag{4a}$$

$$T'_{12} = T'^{*}_{21} = i\delta_d^{-1} \frac{\Delta^2 - 1}{2\Delta} \sin(k_\pm d) \tag{4b}$$

$$T_{11} = T^{*}_{22} = \delta_\pm \left[\cos(k_d d_d) + \frac{i(1+\Delta^2)}{2\Delta} \sin(k_d d_d) \right], \tag{5a}$$

$$T_{12} = T^{*}_{21} = i\delta_\pm^{-1} \frac{1-\Delta^2}{2\Delta} \sin(k_d d_d). \tag{5b}$$

In formulae (3)–(5), $\Delta = \varepsilon_d k_\pm / \varepsilon_a k_d$, $\Delta_{01} = \varepsilon_a k_0 / \varepsilon_0 k_\pm$, $\delta_\pm = \exp(ik_\pm d_a)$ and $\delta_d = \exp(ik_d d_d)$, in which $k_0^2 = (\omega/c)^2$, $k_\pm^2 = (\omega/c)^2 \varepsilon_a \mu_\pm$ and $k_d^2 = (\omega/c)^2 \varepsilon_d$. Equation (2) implies the transmission coefficient

$$t = \frac{1}{\Lambda_{22} + \Lambda_{21}}. \tag{6}$$

The total transmission ratio is $\mathrm{Tr} = |t_+ + t_-|^2/4$, in which t_+ and t_- are the transmissions of the right- and left-handed polarization radiations, respectively. The Faraday rotation angle is shown as

$$\theta = \frac{1}{4(N+1)d_a} \left\{ \arcsin\left(\frac{\mathrm{Im}(t_-)}{|t_-|} \right) - \arcsin\left(\frac{\mathrm{Im}(t_+)}{|t_+|} \right) \right\} + \theta_0, \tag{7}$$

where the rotation angle $\theta_0 = (k_- - k_+)/2$ indicates the normal Faraday effect for the linearly polarized wave through the AF layers or the single AF film. We can conclude that it should be large for frequencies in the vicinity of ω_r as the difference between μ_+ and μ_- is obvious. When $H_0 = 0$, $\mu_+ = \mu_-$, so the rotation vanishes. The first term is an additional rotation caused jointly by such a multilayered structure and the gyromagnetic effect of the AF layers. The next section will show the large Faraday rotation of the defect mode resulting from the diffraction of the right and left circularly polarized waves at the defect layer.

3. Numerical results

For numerical calculations, we consider such an AFPC composed of FeF_2 layers with $\varepsilon_a = 5.5$ and $4\pi M_0 = 7.04$ kG, and SiO_2 layers with $\varepsilon_d = 2.0$. The FeF_2 exchange and anisotropy

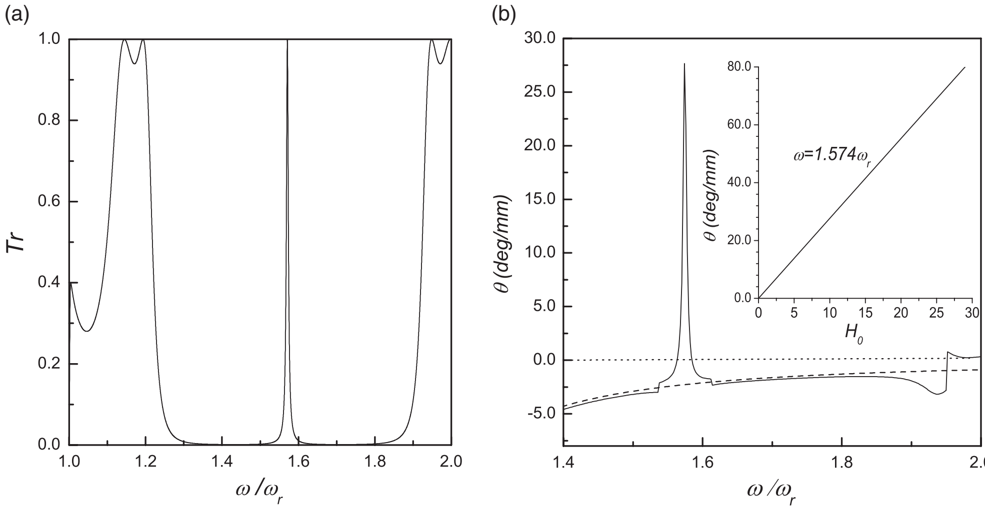

Figure 2. (a) Transmission spectrum of linearly polarized radiation in the range of 52.45 $\mathrm{cm}^{-1} \leqslant \omega/2\pi c \leqslant 104.9\ \mathrm{cm}^{-1}$ and (b) the Faraday rotation in the same range; the inset shows the rotation as a function of the magnetic field H_0. The dashed line corresponds to the relevant single AF film.

fields are taken as $H_e = 533.0$ kG and $H_a = 197.0$ kG, respectively, but the gyromagnetic ratio $\gamma = 1.97 \times 10^{10}$ rad s^{-1} kG^{-1}. Thus the zero-field AF resonance frequency is about $\omega_r = 9.83 \times 10^{12}$ rad s^{-1} ($\omega_r/2\pi c = 52.45\ \mathrm{cm}^{-1}$) and the damping $\tau \approx 0.001\omega_r$. These AF parameters come from [11, 12], where the overall features of both theoretical and experimental results for the AF polaritons of FeF_2 are in very good agreement. $N = 4$ is taken and the AFPC is put in a magnetic field of $H_0 = 10.0$ kG. To keep the defect transmission peaks appearing in the stop bands, we fix the thickness ratio of FeF_2 and SiO_2 layers at $d_a/d_d = \sqrt{\varepsilon_d/\varepsilon_a}$, so that we can take $d_a = 13.0\ \mu$m and $d_d = 21.5\ \mu$m. Only when we discuss the effects of the damping τ and static magnetic field H_0 are the two quantities considered as two variables.

The transmission spectrum of a linearly polarized wave through the AFPC is shown in figure 2(a). From this figure, we see a very sharp peak of transmission of about 100% in the stop band. This peak is the defect mode and it is analogous in physics to an impurity level for semiconductors. Although the AFPC contains just eight bilayers, the transmission spectrum shows a very clear stop band and sharp defect mode transmission. Strictly speaking, two transmission speaks should be seen in this stop band, but the two peaks merge into one, since μ_+ merely differs slightly from μ_- for frequencies in the band. However if one increases H_0 to an appropriate value, this peak must split into two. Figure 2(b) illustrates the Faraday rotation, where a high rotation angle related to the defect mode $\theta = 28^\circ\ \mathrm{mm}^{-1}$ is found. It is about 15 times the size and opposite to that of the single AF film. In addition, this angle increases linearly with H_0, and it is about $\theta = 80^\circ\ \mathrm{mm}^{-1}$ for $H_0 = 30.0$ kG (see the inset that shows the change of the angle with H_0).

Next we discuss the Faraday rotation and transmission in the vicinity of ω_r. Figure 3(a) illustrates the transmission, where the largest transmission in the region between the resonant frequencies ω_1 and ω_2 is about 27% and at $\omega = \omega_r$, with the Faraday rotation of $\theta = 1.7^\circ\ \mu\mathrm{m}^{-1}$. The Faraday rotation in this region is opposite to that outside of it—for example $\theta = -1.0^\circ\ \mu\mathrm{m}^{-1}$ at $\omega = 0.96\omega_r$—but the transmission Tr = 60%. The rotation angle is even larger than that of the defect mode in the stop band, by several numerical orders. The rotation varies sensitively with the external field H_0, as shown in figure 3(c). For $\omega = \omega_r$, the

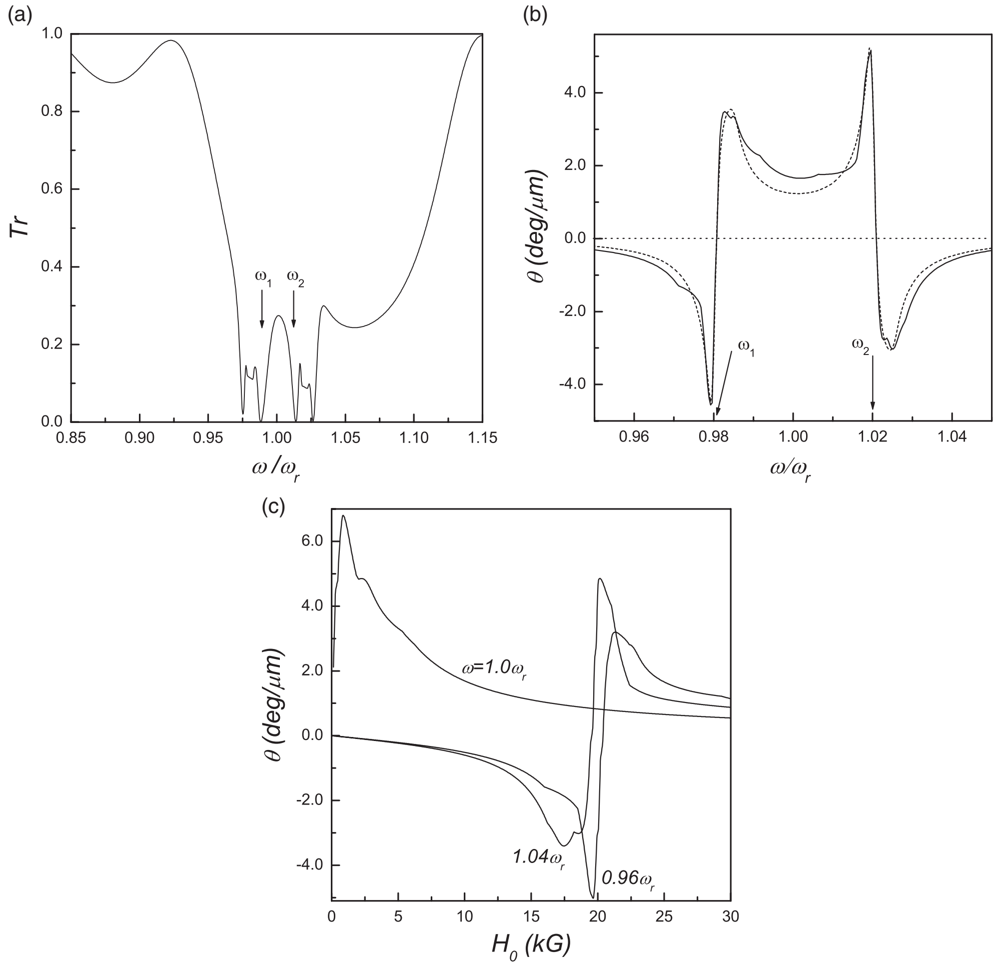

Figure 3. (a) Transmission spectrum of linearly polarized radiation in the vicinity of ω_r, (b) the Faraday rotation of the radiation, where the short dashed curve represents the rotation of the relevant single AF film and (c) the rotation versus the static magnetic field.

largest rotation, $\theta = 7.0^\circ\ \mu\text{m}^{-1}$, appears at about $H_0 = 1.0$ kG, and then the further increase of H_0 leads to a decrease of rotation. For $\omega = 0.96\omega_r$, the negative rotation reaches its largest value $\theta = -5.0^\circ\ \mu\text{m}^{-1}$ at $H_0 = 19.0$ kG; otherwise the positive rotation has its maximum at $H_0 = 21.0$ kG, and then it decreases as the field is further increased from 21 kG. The curve with $\omega = 1.04\omega_r$ shows properties similar to those illustrated by the curve with $\omega = 0.96\omega_r$.

The discussion about the effect of damping is also important, especially in the vicinity of the resonant frequency ω_r. Generally speaking, the resonant absorption from the damping may influence the transmission and rotation, but, as indicated in figure 4, the rotation and transmission decrease only by 20–30% and 20–50%, respectively, as the damping is increased to even 10 times the value $0.001\omega_r$ (about 0.052 cm^{-1}) used in [11, 12].

4. Summary

We find from the numerical results that an AFPC with a defect layer also possesses a high Faraday effect. For the defect mode the Faraday rotation is about 28° mm^{-1}, about 15 times

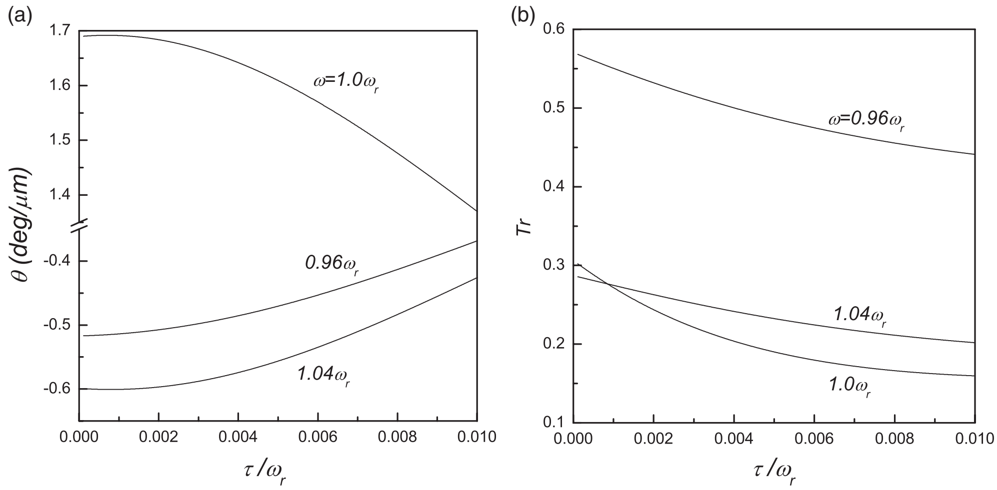

Figure 4. Damping effect on the rotation angle and transmission: (a) the rotation angle versus the damping and (b) the transmission versus the damping.

that of a single AF film. In the vicinity of the zero-field resonance frequency, the rotation angle can be positive or negative, depending on the frequency and external magnetic field, and it is even larger than that of the defect mode: about 250 times. The Faraday effect can be efficiently tuned using the field H_0, including its rotation direction and amplitude. An increase of the damping leads to a decrease of the Faraday rotation and transmission, but it is not very serious. Although this Faraday rotation of the AFPC is smaller for the defect mode than and comparable in the vicinity of ω_r, to that of the ferromagnetic or magneto-optical PCs, the available frequencies are in the far infrared region. The high Faraday rotation of AFPC may be very useful in processing infrared signals. The numerical results are believable, due to the parameters used being determined on the basis of recent experimental results [11, 12].

Acknowledgments

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